

Land Cover projection based on Chain Markov and Cellular Automata: Case study of Pampulha - Brazil

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Abstract

Change models are critical for urban planning practices given their usefulness for generating multiple planning scenarios and evaluating their consequences using a set of metrics or rules. The use of scenarios for projection is not new in the planning activities; however, in many developing countries such as Brazil, this tool is hardly ever used. This paper aims to apply the Change Model technique to project the land cover of Pampulha Regional for 2020 by applying Markov chain and Cellular Automata methodologies. In this article, we present the Pampulha Land Cover maps, the Markovian Probability of Changes maps by categories, the Transition area matrix calculated using Markov Chain and the model validation index. Upon completion of the analysis, we can see that both scenarios have the same trend, in particular, we show that the northwest area of Pampulha will present the greatest changes and will require more attention from the city government of the City of Belo Horizonte. This methodology allows to simulate the future land cover and provide an explanation of the future landscape changes in this part of Brazil. The results are especially important considering a significant role this area plays in the local environmental, urban, architectural and cultural life. The neighborhood has been recognized as an example of modern Brazilian architecture since 1940's, the time when the implementation of urban and architectural Pampulha complex, a unique designs from the architect Oscar Niemeyer occurred. Due to the increased real estate value, the area attracted many investors, which led to a more dynamic landscape conformation. Thus, it's defended that scenario simulation methodologies is a mandatory step in a planning process to be faced by Brazilian municipalities, to make good use of human, environmental and economic resources.

Keywords: Urban Planning; Change Model; Markov Chain, Cellular Automata; Pampulha - Brazil.

1. INTRODUCTION

Brazil has become a predominantly urban country in the 1970s, with the majority of population - 84% - living in cities (IBGE, 2010). If this trend continues into the future, the importance of applying new planning strategies will become more important. With the highest concentration of people and services in urban areas, the issue "urban problems" became obvious. However, it was not given enough attention and wasn't treated as a serious problem by the local authorities. Thus, it is necessary to understand the interaction between different types of urban problems in time and space, and their influence on the environmental, economic and social systems.

One of the methodologies that have highlighted the international scene for studies involving urban dynamics refers to the use of Landscape Dynamic Models. These models constitute one of the best techniques available to satisfy the needs and interests of the Urban Dynamics and regional Land Use investigations (DE ALMEIDA, et al., 2007). The change simulation studies in urban landscapes allow to create future scenarios, provide support to the local governments, guide the decision making process and identify future impacts.

The study methodology is based on two methodologies: the Markov Chain and Cellular Automata. The Markov Chain methodology is used to define the transitional rules. The Cellular Automata is used to project the spatial changes in Land Cover. The study used CA-Markov tool, available in IDRISI Selva software in order to identify the land use trends in 2020. To create the future scenario, we considered that the pattern of land cover in Pampulha will development following the trends of the last 5 years.

2. STUDY AREA – PAMPULHA – BELO HORIZONTE, MINAS GERAIS

The City of Belo Horizonte has been involved in the GIS data research and analysis since 1974, when a separate government institution PRODABEL responsible for GIS analysis was created. In order to demonstrate the potential and limitations of the application of urban simulation studies in Brazil, this paper will use Pampulha Regional (Fig.01), in Belo Horizonte, as a case study.

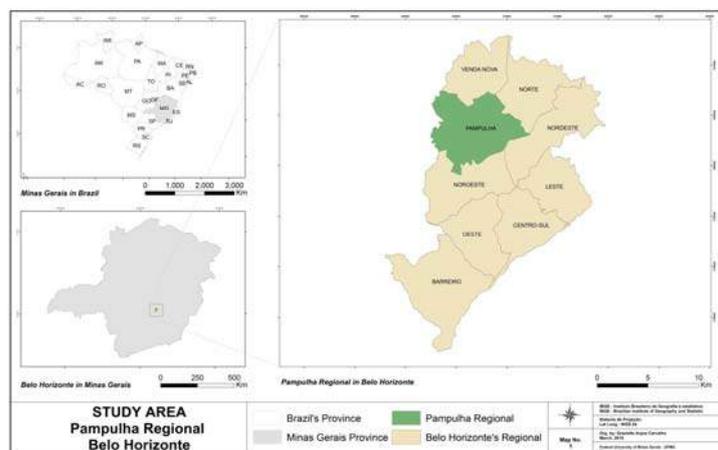


Figure 1: Study area –Pampulha Regional

Due to its environmental characteristics, convenient geographic location and cultural significance, Pampulha has become a popular area in the northern part of Belo Horizonte, Brazil and attracted many private sector investors and real estate professionals. This resulted in a dynamic conformation and landscape transformation. The trend led to a series of environmental, urban, architectural and cultural conflicts which have become crucial in the past several years. The importance of the region and existing problems justify the choice of Pampulha as an object for the current study.

3. MATERIALS

Urban or environmental change modeling can be used to simulate future scenarios and visually represent possible changes in the existing landscape. Design simulation allows to make adjustments to the created model before actual model implementation and minimize negative impacts on the society and the environment.

The basic inputs on landscape change models are the land use or land cover data and the explanatory variables of the object of the study. At the initial data organization stage for this modeling process, data were set considering the following parameters:

- The layers were configured in the same map projection system (UTM - SIRGAS 2000, Zone 23South);
- All layers were in the same data type: raster, with the same numbers of columns and rows; as well as the same extent and cell size (5 meters);
- The data was normalized, which means adjusting values measured on different scales to a notionally common scale; In this normalized scale, low values mean low changes potential

and high values mean high changes potential;

The land cover map was developed with Rapid Eye images (scene 2329919, spatial resolution 5m x 5m), projection WGS84 of Pampulha Regional from 2009 to 2011 and projected from 2013 to 2020. The maps were classified into four categories: Anthropic, Dense Vegetation, Low Vegetation and Water. This was done through a supervised pre-classification (MAXVER), with objects-oriented in the SPRING/INPE - Brazilian software for 2009 and then validated by comparing the automatic classification with visual image interpretation, changing wrongly classified data in the attribute table. Corrections were made for 2009, 2011 and 2013 map classifications. After finalizing this step, the features were reconverted to raster and modeling process was initiated. The maps for 2011 and 2013 were used to calibrate and validate the model.

To analyze the project scenarios, we sought to understand the behavior of the changes through the following explanatory and/or catalysts variables:

- Kernel Density of Commerce activities– By: Cemig
- Pampulha Zoning Ordinances – By: Prodabel/PBH
- Changes potential from vacant lots or single story house plans supply – By: Prodabel/PBH

4. METHODS: MARKOV CHAIN AND CELLULAR AUTOMATA

The Markov Chain method is commonly used with Cellular Automata method. The Markov Chain is usually applied to define the transition rules (probability and area change matrix), the Cellular Automata allows to define the spatial character of the landscape change, considering the initial nearest neighbors of the Land Use/Land Cover.

When the states of the territory distribution is represented discretely (countable), the Markov model is called "Markov Chain". The properties of these models are studied based on transitions matrices properties. Those matrixes are dynamic because it allows to modify the transition probabilities by using a function of time "t", where "t" is discrete or discontinuous (EL YACOUBI, 2006; DIMURO et al., 2002).

According to Soares Filho (1998), Markov Chains are mathematical models used to describe stochastic processes, which means that a family of random variables in a time interval can vary randomly. The Markov Chains can be described the following way:

$$\Pi(t+1) = P^n . \Pi(t) \dots\dots\dots(1)$$

Where $\Pi(t)$ is the actual stage of the system at time t , $\Pi(t+1)$ is the stage of the system at time $t+1$ and P^n are the possible stages to happen, that described at the probability transition matrix. This matrix shows the possibility of a determined stage i remain the same or change for the stage j while the time changes from t to $t+1$.

The transition probabilities usually derived from a sample of transitions occurring during some time interval. A Markov Chain describes a system whose state undergoes a series of changes over time. The changes are not completely predictable, but rather governed by probability distributions. These probability distributions incorporate a simple sort of dependence structure, where the conditional distribution of future states of the system, given some information about past states, depends only on the most recent piece of information. Thus, interactions are instantaneous, being irrelevant to the residence time of the variables in each stage. (SOARES FILHO, 1998; CHANG, 1999).

A Cellular Automata is a cellular entity that independently varies its state based on its previous state and the state of its immediate neighbours according to a specific rule. In the process, only a transition rule is applied which is determined not only by the previous state, but also by the state of the local neighbourhood (BATTY, et al., 1999; BURROUGH, 1998; CÂMARA, 1996; ENGELEN, 1995; EASTMAN, 2012; O’SULLIVAN & TORRENS, 2000; ROY, 1996;).

The Cellular Automata methodology proposes a regular network of identical cells. Each cell can be in one of a finite number of discrete states in discrete time intervals in its evolution. In this model, we start from a random initial state where over time ($T1 \rightarrow T2$) some new cells are born and some die. What determines the state of each cell is its neighbor, which in this case is defined by at least 5x5 adjacent cells. This way, assumes that global spatial patterns emerge from local actions (COUCLELIS, 1985; GEOGRAPHICAL SCIENCES COMMITTEE, 2014; O’SULLIVAN & TORRENS; 2000; PONTIUS JR. et al, 2008; TOBLER, 1979;).

Markov chains and Cellular automata have been used to model changes like spreading of epidemics (MELOTTI, 2009; SASSO, et al., 2004), deforestation (XIMENES et al., 2008; SOARES-FILHO et al., 2003), dynamics of land cover changes in agricultural areas (MACEDO et al., 2013; KAMUSOKO, COURAGE et al., 2009), simulation of traffic and transportation (LIMA, 2007; SUN et al., 2012), dynamics modeling of change in urban areas (ALMEIDA et al., 2003; BATTY, et al 1999; BURROUGH, 1998; CÂMARA, 1996; COUCLELIS, 1997; ENGELEN, 1995; FUGLSANG et al., 2013; MARIA DE ALMEIDA, et al., 2003; MEMARIAN et al., 2012, MITSOVA et al., 2011; ONSTED e CHOWDHURY, 2014; PERES e POLIDORI, 2009; ROY, 1996; VERSTEGEN et al., 2014), population, space and environmental (UMBELINO, G., & BARBIERI, A., 2010), metropolitan urban growth modelling (WHITE e ENGELEN, 1997; UMBELINO, 2012; FURTADO & VAN DELDEN, 2011) and many others.

Softwares or modules that provide this technique are Agent Analyst, DINAMICA, Dynamic Urban Evolution Modeling (DUEM), IDRISI (CellAtom, CA-Markov), Land Transformation Model (LTM), Simulador do Ambiente da Cidade (SACI), Urban Growth Analysis Tool (UGAT), Metronômica, LUCIA (Land Use Change Impact Assessment), or another that can be found in this link: (http://uncomp.uwe.ac.uk/genaro/Cellular_Automata_Repository/Software.html)

In order to apply the model to our study case we used the IDRISI software, CA-Markov tool. The flowchart of this process is presented below. The analysis represented in the flow chart must be done twice (calibration/validation and Future simulation step), in order to get correct results (Figures 2).

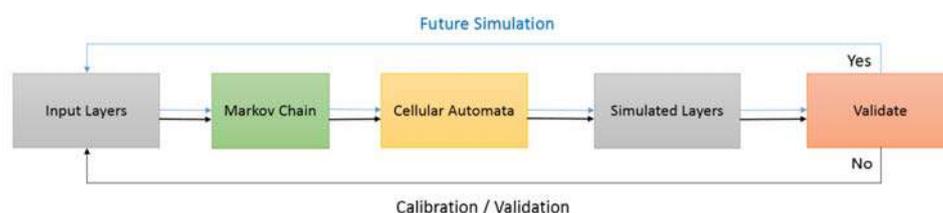


Figure 2: Flowchart of the CA-Markov modelling

The scenarios modelling is a feedback process namely, the output is a parameter that aids in the calibration and validation of the model. At the Markov Chain step (Fig. 3) we have to use two inputs images (land use or land cover images from the same place, but in different time), and the output will be the Probability image change by land cover categories plus the transition probability and transition area matrix.

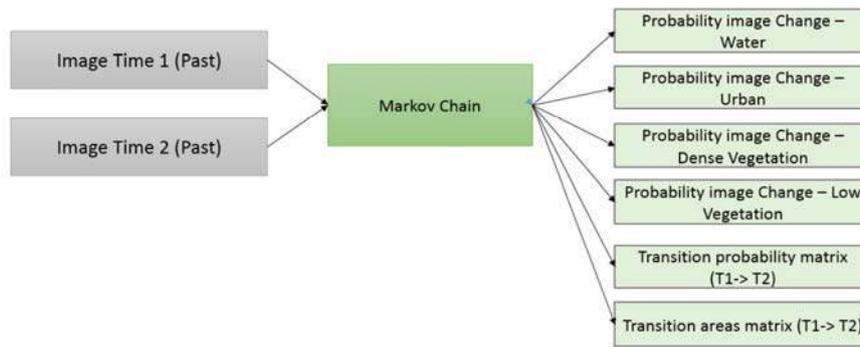


Figure 3: Flowchart of Markov Chain step

Where:

- Image Time 1 = Land Cover of Pampulha at 2009
- Image Time 2 = Land Cover of Pampulha at 2011

The modeling process by Cellular Automata defines the cell change probability based on its immediate neighbours (5x5 filter) and as a result it doesn't use the transition probability matrix created at Markov Chain step.

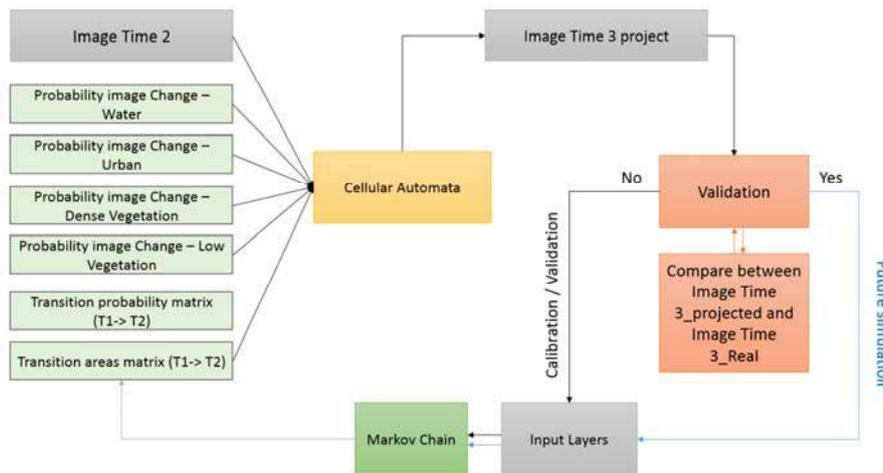


Figure 4: Flowchart of Cellular Automata step

Where:

- Image Time 2 = Land Cover of Pampulha at 2011
- Image Time 3_Real = Real Land Cover of Pampulha at 2013
- Image Time 3_Projected = Projected Land Cover of Pampulha at 2013

The model validation process consists of checking if the constructed model can make predictions correctly. We recommend the use of a validation method that addresses the following topics:

- Identify how much between the two maps did not change: This factor is important to be evaluated when images with a small temporal variation are used or in order to evaluate a low spatial dynamic area. In this conditions we can see that there will be little change between the old map and the current map;
- The model should be able to evaluate the similarity index between the changes and not just the general similarity between the maps: the acceptable value of validation of the maps must be calculated considering the changes and not the general similarity as is currently done by the Kappa index.

Thus, an alternative to the Kappa index is “*Quantity Disagreement*” and “*Allocation Disagreement*”, suggested by Pontius Jr. & Millones (2011). The authors define “*quantity disagreement as the amount of difference between the reference map and a comparison map that is due to the less than perfect match in the proportions of the categories*”. That is, how greater the difference between the number of cells in the same category of cover/use in the reference map (real map) and the projected map, greater will be the “*Quantity Disagreement*”.

Pontius Jr. & Millones (2011) define “*allocation disagreement as the amount of difference between the reference map and a comparison map that is due to the less than optimal match in the spatial allocation of the categories, given the proportions of the categories in the reference and comparison maps*”. In other words, how greater the difference between the location of cells in the same category of cover/use in the reference map (real map) and the projected map, greater will be the “*Allocation Disagreement*”.

The “*Quantity Disagreement*” and “*Allocation Disagreement*”, like as *Kappa* index shows the results in a 0 to 1 scale, where 01 (one) value indicates a validated result and proves the model accuracy. Thus, Pontius Jr. & Millones (2011) calls this index *Kallocation* and *Kquantity*¹.

In order to calculate the *Kallocation* and *Kquantity* index, we used the VALIDATE module in Idrisi. This module offers one comprehensive statistical analysis that answers two important questions at the same time. First question is: How well do a pair of maps agree in terms of the *quantity* of cells in each category? Second question is: How well do a pair of maps agree in terms of the *location* of cells in each category? This module calculates various Kappa Indices of Agreement and related statistics to answer these questions (EASTMAN, 2012) and was based on Pontius (2000, 2011).

5. RESULTS

As results from this paper, we present the Pampulha Land Cover maps (Fig. 5). Each map contains the following categories: Water (blue), Anthropogenic (grey), Dense Vegetation (dark green) and Low Vegetation (light green). Another results are the Markovian Probability of Changes maps by categories (Fig. 6 and 7), the Transition area matrix calculate by Markov Chain (Table 1) and the model validation index (Fig 08), as the Projected Scenarios.

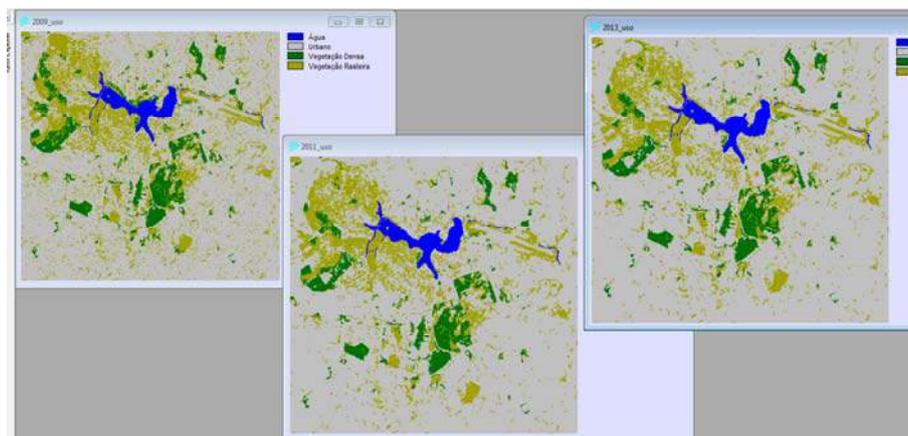


Figure 5: Land Cover of Pampulha at 2009, 2011 and 2013

The Markovian Probability of Changes maps (Fig. 6 and 7) show the potential changes in Land Cover and in a normalized scale (0 to 1), where higher value is symbolized by a darker color and will represent bigger potential changes.

¹ Para maiores informações em como calcular esses índices, ver Pontius e Millones (2011).

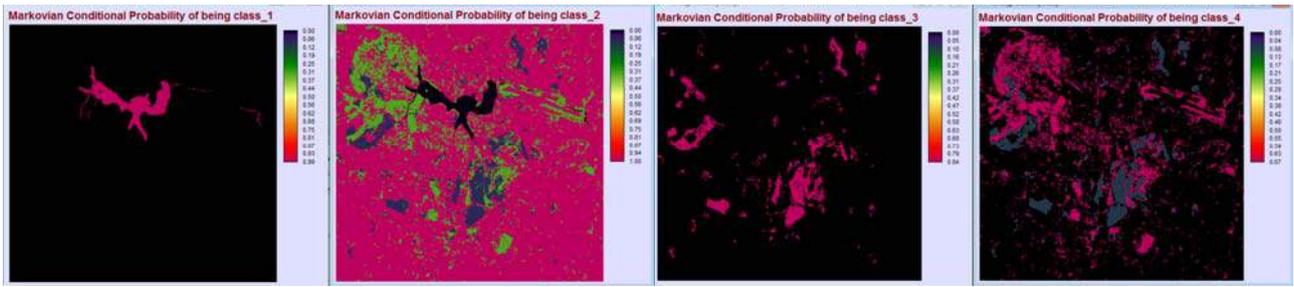


Figure 6: Markovian Probability of Changes maps: Water, Anthropogenic, Dense Vegetation and Low Vegetation

Table 1: Transition area matrix

| Expected to transition to : | | | | |
|-----------------------------|-------|-----------|------------------|----------------|
| | Water | Anthropic | Dense Vegetation | Low Vegetation |
| Water | 81542 | 442 | 0 | 0 |
| Anthropic | 0 | 2831286 | 64 | 0 |
| Dense Vegetation | 0 | 17138 | 170261 | 15364 |
| Low Vegetation | 0 | 167107 | 608 | 342118 |

The validation index for this research is shown in Figure 7:

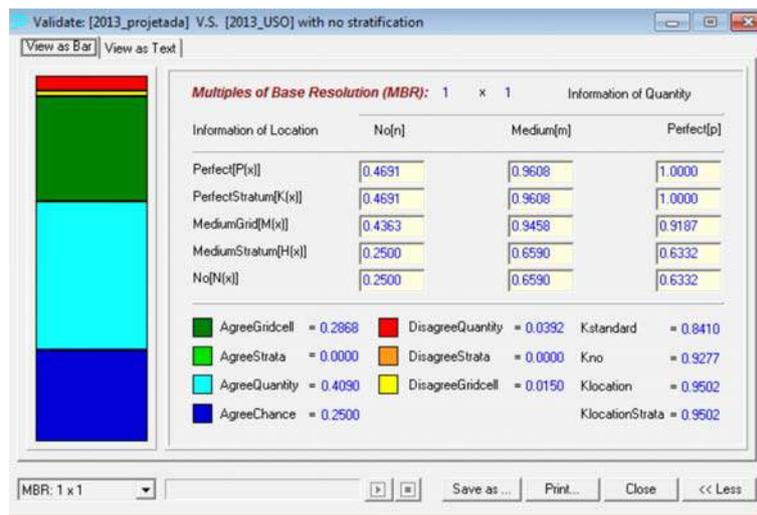


Figure 7: CA-Markov validation index

6. THE PATTERN OF LAND COVER IN PAMPULHA WILL REMAIN AS USUAL IN THE LAST 5 YEARS (2009 – 2013).

A detailed analysis of the real and projected Pampulha's Land Cover shows a great loss of Low Vegetation in northwest portion of the map as well as in the south portion of the Pampulha Lake (Fig. 8).

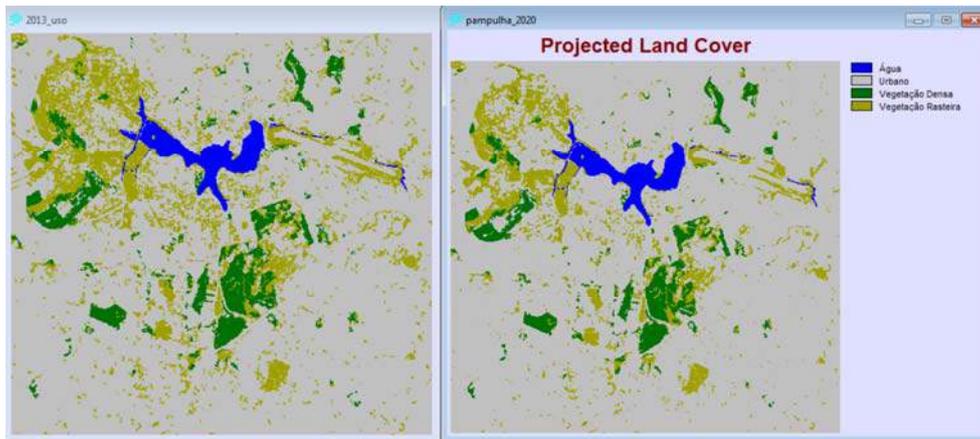


Figure 8: Real (2013) and projected (2020) Land Cover maps of Pampulha

As it is shown in the maps, the large fragments of dense vegetation will remain the same. However, small changes will occur. Low vegetation areas represented by small fragments on the maps, will undergo significant losses. This behavior can be explained by the Landscape Metrics Theory, which explains that larger fragments have a central area which reinforces the whole fragment, making it more stable in the landscape (BORGES, Júnia et al, 2010).

Figure 9 shows that the highest increase is in the category Anthropoc (Urbano) and the lowest is Low Vegetation (Vegetação Rasteira), confirming the map visual analysis.

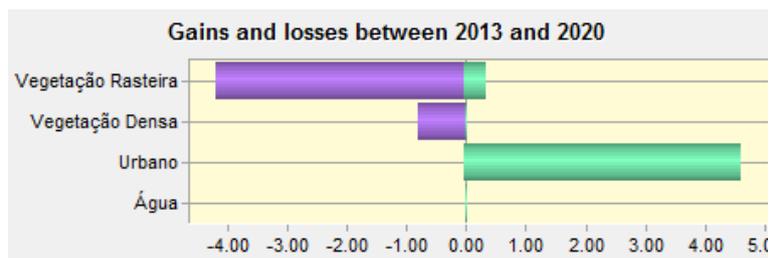


Figure 9: Gains (green) and losses (violet) on the Pampulha's Land cover changes

The analysis of the changes occurring exclusively in Anthropoc demonstrates that the Low Vegetation (Vegetação rasteira) was the category that had the most losses compared to Dense vegetation (Vegetação Densa), which experienced lower level of changes (Fig. 11).

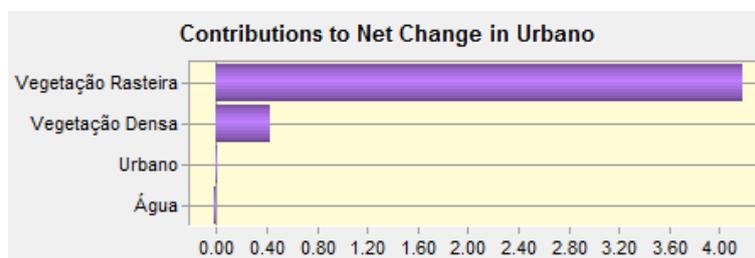


Figure 10: Contributions to net change in Anthropoc

Figure 11a shows the places that will change from any category to Anthropoc in 2020. Once identified the Low Vegetation as the category that most contributes to the Anthropoc expansion in Pampulha, we apply the Cubic trend in that category to identify the transformations hot spot (Fig. 11b). Given this, it appears that the Anthropoc category will increase mainly around the Pampulha Lake and the great loss of Low Vegetation will be in the northwest region of Pampulha. The area will include the following neighborhoods: Bandeirantes, Braúnas, Copacabana, Garças, Jardim Atlântico, Lagoa da Pampulha, Santa Amélia, Trevo e Xangrilá.

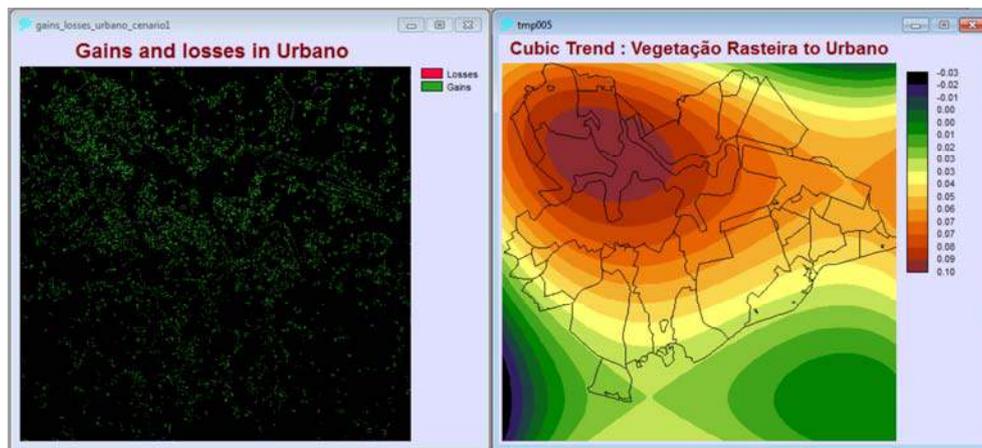


Figure 11: (a) Gains and Losses in Anthropogenic (Urbano) to 2020; (b) Hot spot of the transformations at Pampulha – Scenario 1

7. CONCLUSION

The Pampulha Change Model were performed considering that the pattern of land cover in Pampulha will remain as usual in the last 5 years. In this scenario we could see the northwest area in Pampulha as a trend of the greatest changes and will require more attention from the local government.

This area also has a restrictive zoning ordinance but still has a high potential for changes considering the lot-for-lot capacity utilization. It shows that the results presented in modeling may well come true, which would require preventive actions to promote these scenarios or prevent them from happening.

If the decision to promote a sustainable development in northwestern Pampulha is made, the local government must take measures to increase the basic infrastructure supply. Particularly, introduce more commercial and service centers for the fast growing population.

Despite the urban area be more complex than four land cover categories, this article demonstrates that the use of change models can be an extremely effective tool to support urban planning routines and allows to effectively identify trends.

However, investment in better quality cartographic data (scale, temporality) is critical for the efficient application of the change models in urban planning as well as in many other fields. It will help to make effective planning decisions and modernize the planning process in the urban context by providing greater precision and complexity than is currently available in the Landscape Change Models.

References

1. Almeida, C. M., 2003. Modelagem da dinâmica espacial como uma ferramenta auxiliar ao planejamento: simulação de mudanças de uso da terra em áreas urbanas para as cidades de Bauru e Piracicaba (SP) / C. M. Almeida. – São José dos Campos: INPE, 2003. 351p. – (INPE-10567-TDI/942/A). (accessed 04/03/2013). Available at: http://www.dsr.inpe.br/geu/Teses%20de%20Doutorado/Tese_Claudia_Almeida.pdf
2. Batty, Michael; XIE, Yichun; SUN, Zhanli., 1999. Modeling urban dynamics through GIS-based cellular automata. *Computers, environment and urban systems*, v. 23, n. 3, p. 205-233.

3. Borges, J., Carvalho, G., Moura, A. C. M., & Nascimento, J., 2010. Estudo da conformação da paisagem de Sabará-MG para compreensão das métricas do fragstats em padrões de uso do solo. In: *Congresso Brasileiro de Cartografia*. p. 1473-1481.
4. Burrough, P., 1998. Dynamic Modelling And Geocomputation. In: P. Longley, M. Batty and R. McDonnel. *Geocomputation: A Primer*. London, John Wiley & Sons.
5. Câmara, A. S., 1996. Spatial Simulation Modelling. In: M. Fisher. *Spatial Analytical Perspectives on GIS*. London, Taylor & Francis. Pg. 213-218.
6. Chang, J., 1999. Markov Chain – Stochastic Processes. Available at: http://www.isn.ucsd.edu/classes/beng260/complab/week2/Chang_1999.pdf
7. Couclelis, H., 1985. Cellular worlds: a framework for modelling micro-macro dynamics. *Environment and Planning A*, 17, 585-596.
8. Couclelis, H. From Cellular Automata to Urban Models: New Principles for Model Development and Implementation. *Environment and Planning B: Planning and Design* 24: (1997). 165-174.
9. De Almeida, Cláudia Maria; Câmara, Gilberto; Monteiro, Antônio Miguel Vieira, 2007. Geoinformação em urbanismo: cidade real× cidade virtual. *Oficina de Textos*.
10. Dimuro, Graçaliz P. et al., 2002. Modelos de Markov e Aplicações. *VI Oficina de Inteligência Artificial*, Pelotas: Educat, p. 37-59.
11. Eastman, J. R., 2012. IDRISI Selva Manual. *Clark University*. Sitio web: www.clarklabs.com.
12. El Yacoubi, Samira; Chopard, Bastien; Bandini, Stafania., 2006. Cellular Automata: 7th International Conference on Cellular Automata for Research and Industry, ACRI 2006, Perpignan, France, September 20-23, 2006, Proceedings. Springer, 2006.
13. Engelen, Guy et al., 1995. Using cellular automata for integrated modelling of socio-environmental systems. *Environmental monitoring and Assessment*, v. 34, n. 2, p. 203-214.
14. Fuglsang, Morten; Münier, Bernd; Hansen, Henning Sten., 2013. Modelling land-use effects of future using cellular automata: An Eastern Danish case. urbanization *Environmental Modelling & Software*, v. 50, p. 1-11, 2013.
15. Furtado, Bernardo Alves; Van Delden., 2011. Hedwig. Modelagem urbana e regional com autômatos celulares e agentes: panorama teórico, aplicações e política pública. *Instituto de Pesquisas Econômica e aplicada - IPEA*. (accessed 2013, May). Available at: http://www.ipea.gov.br/sites/000/2/publicacoes/tds/td2_1576_Web.pdf
16. Geographical Sciences Committee, et al., 2014. Advancing Land Change Modeling: Opportunities and Research Requirements. *National Academies Press*.
17. IBGE, 2010. Census. Available at www.ibge.gov.br
18. Kamusoko, Courage et al., 2009. Rural sustainability under threat in Zimbabwe—simulation of future land use/cover changes in the Bindura district based on the Markov-cellular automata model. *Applied Geography*, v. 29, n. 3, p. 435-447.
19. Lima, Edgar Barbosa. 2007 Modelos microscópicos para simulação do tráfego baseados em autômatos celulares. Tese de Doutorado. PHD thesis, Universidade Federal Fluminense.
20. Maria De Almeida, Cláudia et al, 2003. Stochastic cellular automata modeling of urban land use dynamics: empirical development and estimation. *Computers, Environment and Urban Systems*, v. 27, n. 5, p. 481-509, 2003.
21. Macedo, Rodrigo de Campos, et al. Modelagem dinâmica espacial das alterações de cobertura e uso da terra relacionadas à expansão canavieira. *Boletim de Ciências Geodésicas*, sec. Artigos, v. 19, n. 2, p. 313-337, 2013.
22. Melotti, Gledson., 2009. Aplicação de Autômatos Celulares em Sistemas Complexos: Um Estudo de Caso em Espalhamento de Epidemias. Tese de Doutorado. Universidade Federal de Minas Gerais.
23. Memarian, Hadi et al., 2012. Validation of CA-Markov for simulation of land use and cover change in the Langat Basin, Malaysia. *Journal of Geographic Information System*, v. 4, p. 542.

24. Mitsova, Diana; Shuster, William; WANG, Xinhao., 2011. A cellular automata model of land cover change to integrate urban growth with open space conservation. *Landscape and Urban Planning*, v. 99, n. 2, p. 141-153.
25. O'sullivan, David; Torrens, Paul M., 2001. Cellular models of urban systems. In: *Theory and Practical Issues on Cellular Automata*. Springer London, p. 108-116.
26. Onsted, Jeffrey A.; Chowdhury, Rinku Roy., 2014. Does zoning matter? A comparative analysis of landscape change in Redland, Florida using cellular automata. *Landscape and Urban Planning*, v. 121, p. 1-18.
27. Peres, O. M. & Polidori, M. C., 2009. Modelos Urbanos baseados em Autômatos Celulares: integrando ambiente natural e o crescimento urbano. (accessed 2013, March). Available at: <http://www.lbd.dcc.ufmg.br/colecoes/wcama/2009/010.pdf>
28. Pontius, R.G. , 2000. Quantification error versus location error in comparison of categorical maps. *Photogrammetric Engineering & Remote Sensing*, 66(8): 1011-1016.
29. Pontius Jr, Robert Gilmore et al. , 2008. Comparing the input, output, and validation maps for several models of land change. *The Annals of Regional Science*, v. 42, n. 1, p. 11-37.
30. Pontius Jr, Robert Gilmore & Millones, Marco.; 2011. Death to Kappa: birth of quantity disagreement and allocation disagreement for accuracy assessment, *International Journal of Remote Sensing*, 32:15, 4407-4429.
31. Prefeitura de Belo Horizonte., 200-. Pampulha: modernização urbanística acelerada. (Accessed at 2015, January). Available at: <http://portalpbh.pbh.gov.br/pbh/>
32. Roy, Geoffrey G.; Snickars, Folke.,1996. City life; a study of cellular automata in Urban Dynamics. *Spatial Analytical Perspectives and GIS*, Taylor and Francis, London, p. 213-229.
33. Sasso, Juliana Bonás; Chimara, Henrique Dal Bo; Monteiro, Luiz Henrique Alves. Epidemias e modelos epidemiológicos baseados em autômatos celulares: uma breve revisão. *Cad. de Pós-Graduação em Eng. Elétrica*, v. 4, n. 1, p. 71-8, 2004.
34. Soares Filho, Britaldo Silveira, 1998. Modelagem da dinâmica de paisagem de uma região de fronteira de colonização amazônica. *Tese de Doutorado*. Universidade de São Paulo.
35. Soares-Filho, Britaldo Silveira, et al., 2003. Modelagem de dinâmica de paisagem: concepção e potencial de aplicação de modelos de simulação baseados em autômato celular. Ferramentas para modelagem da distribuição de espécies em ambientes tropicais. Belém (Available at: www.csr.ufmg.br/dinamica).
36. Sun, Jian et al. , 2012. CA-Based Urban Land Use Prediction Model: A Case Study on Orange County, Florida, US. *Journal of Transportation Systems Engineering and Information Technology*, v. 12, n. 6, p. 85-92.
37. Tobler, W. Cellular Geography. In Gale, S. & Olssen, G. (Eds.) *Philosophy in Geography*. Dordrecht, Reidel, 1979.
38. Umbelino, G. Barbieri, A. Uso de Autômatos Celulares em estudos de população, espaço e ambiente. *XVII Encontro Nacional de Estudos Populacionais – ABEP – Caxambu – 20 a 24 de Setembro de 2010*. Accessed 2013, March. Available at: http://www.abep.nepo.unicamp.br/encontro2010/docs_pdf/eixo_2/abep2010_2432.pdf
39. Umbelino, G. J.M.,2012. Simulações de distribuição espacial domiciliar e projeções demográficas intraurbanas com apoio de geotecnologias. *Tese de Doutorado* - Universidade Federal de Minas Gerais, Programa de Pós Graduação em Demografia, Belo Horizonte, 2012.
40. Verstegen, Judith A. et al.,2014. Identifying a land use change cellular automaton by Bayesian data assimilation. *Environmental Modelling & Software*, v. 53, p. 121-136.
41. White R; Engelen G.,1997. Cellular automata as the basis of integrated dynamic regional modeling. *Environment and Planning B: Planning and Design*, 24(2): 235–246.
42. Ximenes, A. de C. et al.,2008. Modelagem Dinâmica do desmatamento na Amazônia. *Boletim de Ciências Geodésicas*, v. 14, n. 3, p. 370-391.